First Day. Tuesday Octuber 4th, 2011.

1. (Universidad de los Andes) Find all real numbers a for which there exist distinct real numbers b, c, d different from a such that the four tangents to the curve $y = \sin(x)$ at the points $(a, \sin(a))$, $(b, \sin(b))$, $(c, \sin(c))$ y $(d, (d))$ form a rectangle.

2. (Brazilian Mathematics Olympiad) Let k be a positive integer and let a be an integer such that $a-2$ is a multiple of 7 and a^6-1 is a multiple of 7^k. Show that $(a+1)^6-1$ is also a multiple of 7^k.

3. (Geza Kos) Let $f(x)$ be a rational function racional with complez coefficients whose denominator doesn't have multiple roots. Let $u_0, u_1, \ldots, u_n$ be the complex roots of f and $w_1, w_2, \ldots, w_m$ be the roots of f'. (Each root is considered as many times as its multiplicity). Suppose that u_0 is a single root of f. Show that

$$\sum_{k=1}^{m} \frac{1}{w_k - u_0} = 2 \sum_{k=1}^{n} \frac{1}{u_k - u_0} .$$

   Note: A rational function is the quotient of two polynomials.